Option Valuation for Lévy Innovation GARCH Models under Sequential Bayesian Learning

Fumin Zhu

Key Lab of FIFE
at Southwestern University of Finance & Economics
Visiting scholar in Quantitative Finance Club
Department of Applied Mathematics and Statistics
at Stony Brook University

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Outline

1. Introduction
2. Dynamic Model
3. Econometric Methodology
4. Option Valuation
5. Empirical Research
6. Conclusion and Summary
Introduction

Dynamic Model

Econometric Methodology

Option Valuation

Empirical Research

Conclusion and Summary
Introduction: my Models and Methods

- Lévy innovation GARCH models (LIGM)
  - Asymmetric GARCH models with discrete-time Lévy processes;
  - Features: Dynamic volatility, Leverage effect, Non-normality;
  - Easy Estimation: Numerical MLE(jointly, separately).

- Sequential Bayesian learning approach (SBLA)
  - A simulated-based filtering technique;
  - Perfect for Dynamic State Space and latent states filtering;
  - The drift, volatility, jumps as conditional state variables;

My work

Transform the LIGM into a states space structure; construct a new dynamic discrete-time Lévy process; Estimate the parameter and states jointly by SBLA.
Introduction: Contributions and findings

- Risk neutral pricing model
  - Risk neutral valuation relationship: obtain its EMMs.
  - The distribution of neutralized Lévy innovations.
  - The conditional states (volatility) for Neutralized GARCH dynamics.

- Econometric methodology
  - Numerical maximum likelihood estimation (batch estimator).
  - Sequential Bayesian learning approach (sequentially).
  - The performance on return rates and options.

My finding

SBLA decreases 25% of the IV-RMSE on average in-sample, 20% out of sample with respect to NMLE benchmark NGARCH models.
1 Introduction

2 Dynamic Model

3 Econometric Methodology

4 Option Valuation

5 Empirical Research

6 Conclusion and Summary
Benchmark model

Benchmark conditional model

\[ y_t = c_t + \varepsilon_t, \varepsilon_t = \sigma_t z_t, \quad (1) \]
\[ c_t = \mathbb{E}[y_t|\mathcal{F}_{t-1}], \quad \sigma_t^2 = \mathbb{E}[y_t^2|\mathcal{F}_{t-1}] \quad (2) \]
\[ \mathbb{E}[z_t|\mathcal{F}_{t-1}] = 0, \quad \mathbb{E}[z_t^2|\mathcal{F}_{t-1}] = 1 \quad (3) \]

where \( y_t \equiv \log(S_t/S_{t-1}) \) are log return rates, \( c_t \) the conditional expectation, \( \sigma_t^2 \) the conditional variance, \( \varepsilon_t \) innovations and \( z_t \) standard innovations. Here, \( \mathcal{F}_t \) represents the available information at time \( t \).
Conditional dynamics follow ARMA and N-GARCH

\[
\begin{align*}
  c_t &= \mathbb{E}[y_t|\mathcal{F}_{t-1}] = c + a\, y_{t-1} + b\, \varepsilon_{t-1} \quad (4) \\
  \sigma_t^2 &= \alpha_0 + \alpha_1(\varepsilon_{t-1} - \gamma\sigma_{t-1})^2 + \beta\sigma_{t-1}^2 \quad (5)
\end{align*}
\]

where \( a, b, c, \alpha_0, \alpha_1, \beta, \gamma \) are parameters. \( \gamma = 0 \) represents classical symmetric GARCH model, \( \gamma > 0 \) can express the leverage effect (negative relationship).
Conditional State variables and state space

- Conditional drift, volatility and innovations

\[ \mu_t = c^*_t + a \mu_{t-1} + b^* \varepsilon_{t-1} \]  
\[ \sigma^2_t = \alpha_0 + \alpha_1 \sigma^2_{t-1} (z_{t-1} - \gamma)^2 + \beta \sigma^2_{t-1} \]  
\[ \varepsilon_t = \sigma_t z_t, \quad z_t = \Delta X_t \sim (x; m, \delta, \nu(dx)) \]  
\[ b^* = (a + b), \quad c^*_t = c + \varphi(\sigma_t) - a \varphi(\sigma_{t-1}) \]

where \( \mu_t = \log \mathbb{E}[e^{y_t}|\mathcal{F}_{t-1}] \) is the exponential drift. \( \varphi(u) = \log \mathbb{E}[e^{uz_t}] \) expresses the MGE function. \( X_t \) is a Lévy process, \( \Delta X_t \) is its standard increment, following \( \varphi'_{\Delta X_t}(0) = 0 \) and \( \varphi''_{\Delta X_t}(0) = 1 \).
Comparing: A discrete-time dynamic Lévy process

Supposing the innovation follows

$$\varepsilon_t = \Delta L_t = x_t \Delta X_t$$  \hspace{1cm} (10)

assume its conditional states come from $$x_t \subset \{ \mu_t, \sigma_t, h_t, \ldots \}$$, and stochastic factors $$\Delta X_t \subset \{ \Delta_t, \Delta W_t, \Delta J_t, \ldots \}$$.

Its MGE of $$\Delta L_t$$ and mean-correction

$$\varphi_{\Delta L_t}(u|x_t) = \varphi_{\Delta X_t}(x_t, u)$$  \hspace{1cm} (11)

$$\log \mathbb{E}[e^{\Delta L_t}|x_t] = \varphi_{\Delta X_t}(x_t)$$  \hspace{1cm} (12)

Notice, $$x_t$$ expresses the conditional states of the Lévy process, which is determined by its former information $$\mathcal{F}_{t-1}$$, independent with $$\Delta X_t$$. 

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Option Valuation for Lévy GARCH Models With SBLA
Dynamic state space for LIGM

- LIGM belongs to one stochastic factor state structure

\[ y_t = \mu_t - \varphi \Delta X_t(x_t) + x_t \Delta X_t \]  \hspace{1cm} (13)

\[ \mu_t = \log \mathbb{E}[e^{y_t}|\mathcal{F}_{t-1}] \]  \hspace{1cm} (14)

\[ x_t^2 = \alpha_0 + \alpha_1 x_{t-1}^2 (\Delta X_{t-1} - \gamma)^2 + \beta x_{t-1}^2 \]  \hspace{1cm} (15)

\[ z_t = \Delta X_t \sim (x; m, \delta, v(dx)) \]  \hspace{1cm} (16)

where \( x_t = \sigma_t \) expresses the conditional volatility of the return rates series, or \( x_t = h_t \) means conditional intensities for jumps, both determined by former information \( \mathcal{F}_{t-1} \).
A fully adapted dynamic state space

Multiple stochastic factors

\[ x_t = [\mu_t, \sigma_t, h_t, \ldots] \]  
\[ \Delta X_t = [\Delta_t, \Delta W_t, \Delta J_t, \ldots]^T \]  
\[ \Delta L_t = x_t \Delta X_t, \varepsilon_t = \Delta L_t \]  
\[ x_t = G(\Delta X_{t-1}, x_{t-1}, \theta_G) \]

where \( \theta_G \) is the parameters in conditional transition function. \( x_t \) describes conditional states, \( \Delta X_t \) expresses the stochastic factors separately for business time, diffusion and jumps.
1. Introduction

2. Dynamic Model

3. Econometric Methodology

4. Option Valuation

5. Empirical Research

6. Conclusion and Summary
Numerical method: Quasi-MLE

- Two moments, Quasi-MLE
- Two-step estimation, Log-likelihood:

\[
llf(y_{1:T}; \Theta) = \sum_{t=1}^{T} \left\{ -\frac{(y_t - c_t)^2}{2\sigma_t^2} + \log \frac{1}{\sqrt{2\pi\sigma_t}} \right\}
\]

\[
\hat{\Theta} = \text{arg max } llf(y_{1:T}; \Theta)
\]

\[
\hat{\Theta}_z = \text{arg max } f(z_{1:T}; \Theta_z)
\]

Two-step estimation

Where \( c_t, \sigma_t \) refers to the first and second moment (conditionally), use QMLE to obtain the standard innovations and separately estimate it by FFT.
Estimation: Innovations’ density

- One-step estimation
- Conditional density of Lévy innovations (joint, and FFT)

\[
f(\varepsilon_t | \sigma_t) = \frac{1}{2\pi \sigma_t} \int_{-\infty}^{\infty} e^{-izt\sigma_t u} \phi_{\zeta_t}(\sigma_t u) d(\sigma_t u) \tag{24}\]

\[
f(\Delta L_t | x_t) = f(x_t \Delta X_t) = \frac{f(\Delta X_t)}{x_t} \tag{25}\]

\[
f(\Delta X_t) = f(z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-iuz} \phi_z(u) du \tag{26}\]

where \( \phi(u) \) refers to the characteristic function. \( f(\Delta L_t) \) is the joint density, \( x_t \) is just a constant value at time \( t \).
Numerical method: Joint MLE

- Log-likelihood: joint density for $\varepsilon_t$

$$lhf(y_{1:T}; \Theta) = \sum_{t=1}^{T} \{\log f(\varepsilon_t|x_t)\}$$

(27)

$$\log f(\varepsilon_t|x_t) = \log f(\frac{\varepsilon_t}{x_t}) - \log x_t$$

(28)

$$\hat{\Theta} = \arg \max \{lhf(y_{1:T}; \Theta)\}$$

(29)

One-step estimation

Where $x_t, \Delta X_t$ refers to conditional states and stochastic factor, the innovations are estimated by FFT.
A Monte Carlo method: particle filtering

- Suppose we have some prior information $F_{t-1}$
- Sampling particles $x_{t-1}^{(i)}, \Delta X_{t-1}^{(i)}$ and estimate $p(\Delta X_t^{(i)})$

$$p(y_t|y_{t-1}; \Theta) \approx \frac{1}{N} \sum_{i=1}^{N} p(y_t|x_t^{(i)}; \Theta)$$  \hspace{1cm} (30)

$$w_t^{(i)} = p(y_t|x_t^{(i)}; \Theta) = \frac{p(\Delta X_t^{(i)}; \Theta)}{x_t^{(i)}}$$  \hspace{1cm} (31)

$$llf(y_{1:T}; \Theta) \approx \sum_{t=1}^{T} \log \left\{ \frac{1}{N} \sum_{i=1}^{N} w_t^{(i)} \right\}$$  \hspace{1cm} (32)

- You can maximize the likelihood: $\hat{\Theta} = \text{arg max} \{llf(y_{1:T}; \Theta)\}$
Sequential Bayesian Learning Approach (SBLA)

- Based on Bayesian rule, the posterior density for $\Theta$

$$p(\Theta | y_{1:T}) \propto p(y_{1:T} | \Theta)p(\Theta)$$  \hspace{1cm} (33)

- Sampling $\Theta^{(m)}$, its importance weights

$$w(\Theta^{(m)}) \propto p(y_{1:T} | \Theta^{(m)})p(\Theta^{(m)})$$  \hspace{1cm} (34)

$$\tilde{w}(\Theta^{(m)}) = w(\Theta^{(m)})/\sum_{n=1}^{M} w(\Theta^{(n)})$$  \hspace{1cm} (35)

- Now, the joint distribution for $\{\Theta^{(m)}, x_t\}$

$$p(\Theta^{(m)}, x_{0:T} | y_{1:T}) = p(x_{0:T} | y_{1:T}, \Theta^{(m)}) \tilde{w}(\Theta^{(m)})$$  \hspace{1cm} (36)
The mean and covariance for parameters

\[ \hat{\Theta} = \sum_{m=1}^{M} \Theta^{(m)} \tilde{w}(\Theta^{(m)}) \]  

(37)

\[ \hat{\Sigma} = \sum_{m=1}^{M} (\Theta^{(m)} - \hat{\Theta}) \tilde{w}(\Theta^{(m)}) (\Theta^{(m)} - \hat{\Theta}) \]  

(38)

Specially, we only drive MCMC when efficient particles degenerating.

Now, parameter updated with acceptance rates

\[ \alpha(\Theta^*) = 1 \wedge \frac{p(\Theta^*) p(y_{1:T} | \Theta^*) N(\Theta; \hat{\Theta}^*, \hat{\Sigma})}{p(\Theta) p(y_{1:T} | \Theta) N(\Theta^*; \hat{\Theta}, \hat{\Sigma})} \]  

(39)
Difference, NMLE (static) and SBLA (dynamic)

- SBLA uses the prior information of the parameters $p(\Theta)$ at each $t$

  \[
p(\Theta^{(m)}) \propto p(y_{1:t}|\Theta^{(m)}) \quad (40)
  \]

  \[
p(\Theta^{(m)}, x_0:T|y_{1:T}) = p(x_0:T|y_{1:T}, \Theta^{(m)})p(\Theta^{(m)}) \quad (41)
  \]

- SBLA sequentially updates the model when new observation comes up

  \[
  w_t^{(i)} = p(y_t|x_t^{(i)}; \Theta) = \frac{p(\Delta X_t^{(i)}; \Theta)}{x_t^{(i)}} \quad (42)
  \]
1. **Introduction**

2. **Dynamic Model**

3. **Econometric Methodology**

4. **Option Valuation**

5. **Empirical Research**

6. **Conclusion and Summary**
Risk neutral pricing model(1): Local EMMs

- Lévy Measure transformation

- \( Q, P \) have triplets \((m^*, \sigma^*, \nu^*)\) and \((m, \sigma, \nu)\)

\[
\sigma^* = \sigma \tag{43}
\]

\[
m^* - m = \int_{-\infty}^{\infty} x(\nu^* - \nu)(dx) + \kappa \sigma^2 \tag{44}
\]

where \( \kappa \) is constant if \( \sigma > 0 \) and zero if \( \sigma = 0 \). \( Q, P \) are equivalent. Sato (1999), Cont and Tankov (2004) and Bianchi, Rachev and Fabozzi (2012).
Risk neutral pricing model(1): GARCH dynamics

- Risk neutral GARCH Transformation

\[ y_t = r_t - \tilde{\phi}(\sigma_t) + \sigma_t \tilde{z}_t, \quad \tilde{z}_t = z_t + k_t \] \hspace{1cm} (45)

\[ k_t = \lambda_t + \frac{\tilde{\phi}(\sigma_t) - \varphi(\sigma_t)}{\sigma_t}, \quad \lambda_t = \frac{\mu_t - r_t}{\sigma_t} \] \hspace{1cm} (46)

- About std.TS: \( \tilde{\varphi}'(0) = 0, \tilde{\varphi}''(0) = 1 \)

\[ C^* = C, \quad \alpha^* = \alpha, \quad m^* + k_t = m \] \hspace{1cm} (47)

where \( \tilde{\varphi}(u) \) is the equivalent risk neutral MGE function. See Kim, Bianchi, Rachev and Fabozzi from 2008 to 2012. The volatility keeps unchanged.
Risk Neutral pricing kernel(2): RNVR

- Risk neutral valuation relationship (RNVR)
- Pricing kernel: Radon-Nikodym derivatives.

\[
\xi \Delta X_t(\theta_t)|\mathcal{F}_{t-1} = e^{-\theta_t \Delta X_t - \phi \Delta X_t(-\theta_t)}
\]

\[
E[\xi \Delta X_t(\theta_t)|\mathcal{F}_{t-1}] = 1
\]

- RNVR transformation.

\[
E^Q_t[e^{y_t}] = E[e^{y_t}|\mathcal{F}_{t-1}\xi \Delta X_t(\theta_t)] = e^{r_t}
\]

\[
\log E^Q_t[e^{u \Delta X_t}] = \log E[e^{u \Delta X_t} \cdot \xi \Delta X_t(\theta_t)]
\]

where \(\theta_t\) is the EMMs coefficient acting as the wedge between two measures of interest.
Risk Neutral EMMs under RNVR

- RNVR MGE function.

\[
\varphi_{\Delta X_t}^Q(u) = \varphi_{\Delta X_t}(u - \theta_t) - \varphi_{\Delta X_t}(-\theta_t)
\]  

(52)

- Constrained condition for EMMs coefficients.

\[
\varphi_{\Delta X_t}(x_t) + \varphi_{\Delta X_t}(-\theta_t) - \varphi_{\Delta X_t}(x_t - \theta_t) = \lambda_t x_t
\]  

(53)

- Moments of RNVR EMMs.

\[
\varphi_{\Delta X_t}^{Q_t}(0) = \varphi_{\Delta X_t}^{(n)}(-\theta_t)
\]  

(54)
RNVR EMMs distribution

- Use Taylor expansion to analyze its distribution.
- With $\theta_t > 0$ and left-skewness.

$$
\varphi_{\Delta X_t}(-\theta_t) < \varphi_{\Delta X_t}(0) = 0 \quad (55)
$$

$$
\varphi_{\Delta X_t}''(-\theta_t) > \varphi_{\Delta X_t}''(0) = 1 \quad (56)
$$

$$
\varphi_{\Delta X_t}^{(3)}(-\theta_t) < \varphi_{\Delta X_t}^{(3)}(0) < 0 \quad (57)
$$

$$
\varphi_{\Delta X_t}^{(4)}(-\theta_t) > \varphi_{\Delta X_t}^{(4)}(0) > 0 \quad (58)
$$

Non-Gaussian Framework

RNVR EMM presents more non-normality.
RNVR GARCH dynamics

- Standardize the RNVR stochastic factor.
\[
\Delta X_t^* = \frac{\Delta X_t - \varphi'_\Delta X_t(-\theta_t)}{\sqrt{\varphi''_{\Delta X_t}(-\theta_t)}}
\]  (59)

- RN GARCH transformation.
\[
x_t^* = \sqrt{\varphi''_{\Delta X_t}(-\theta_t)} \cdot G(\Delta X_{t-1}, \frac{x_{t-1}^*}{\sqrt{\varphi''_{\Delta X_t}(-\theta_{t-1})}})
\]  (60)
Standardize the RNVR stochastic factor.

\[ h_t^* = \alpha_0^* + \alpha_1^* h_{t-1}^* (z_{t-1}^* - \gamma^*)^2 + \beta^* h_{t-1}^* \quad (61) \]

Parameter transformation.

\[ \alpha_0^* = \varphi''(-\theta_t)\alpha_0, \alpha_1^* = \alpha_1 \varphi''(-\theta_t) \quad (62) \]

\[ \gamma^* = \frac{\gamma - \varphi'(-\theta_t)}{\sqrt{\varphi''(-\theta_t)}}, \beta^* = \beta \quad (63) \]
### Transformation for Lévy process

#### Table: Relationship

<table>
<thead>
<tr>
<th>ModEMMs</th>
<th>Neutralized Moments</th>
</tr>
</thead>
<tbody>
<tr>
<td>BS</td>
<td>$\mu^* = \mu - \theta \sigma^2$, $\sigma^* = \sigma$</td>
</tr>
<tr>
<td>MJ</td>
<td>$\lambda^* = \lambda e^{-\mu^<em>_j \theta + \frac{\sigma^2 \theta^2}{2}}$, $\mu^</em>_j = \mu_j - \theta \sigma^2_j$, $\sigma^*_j = \sigma_j$</td>
</tr>
<tr>
<td>VG</td>
<td>$M^* = M + \theta$; $G^* = G - \theta$;</td>
</tr>
<tr>
<td>CTS</td>
<td>$\lambda^<em><em>+ = \lambda</em>+ + \theta_t$, $\lambda^</em><em>- = \lambda</em>- - \theta_t$, $\alpha^* = \alpha$</td>
</tr>
</tbody>
</table>
1 Introduction

2 Dynamic Model

3 Econometric Methodology

4 Option Valuation

5 Empirical Research

6 Conclusion and Summary
Model, method and data selection

- **Model**: Four kinds of Lévy process (BS, MJ, VG, and CTS). ARMA effect and N-GARCH dynamics for drift and volatility. One stochastic factor for fair comparison to the benchmark model.

- **Data**: S&P 500 return rates and options (implied volatility). Daily closed prices from Jan 1995 to Nov 2012, a total number of 4654. We use 9 moneyness options with 1 month maturity because of active trading, that's 17757 options, covering from Jan 3, 2005 to Oct 31, 2012.

- **Method**: RNVR, NMLE and SBLA. The standard innovations play a role of observation noise, so we fix the Lévy measures in the whole sample space and just learning the 5 parameters in N-GARCH dynamics in order to estimate these time changed state variables more efficiently and fairly. Under this treatment, each model has the same number of learning parameters equally.
# Time Series Analysis (QMLE)

**Table:** The Descriptive Statistics of SP500 return rates

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>St.Dev</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>Min</th>
<th>Max</th>
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</thead>
<tbody>
<tr>
<td>$y_t$</td>
<td>2.43E-4</td>
<td>0.0124</td>
<td>-0.2326</td>
<td>11.0217</td>
<td>-0.0947</td>
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<td>$z_t$</td>
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<td>1.0116</td>
<td>-0.4279</td>
<td>4.3672</td>
<td>-6.1581</td>
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<tr>
<td>$\sigma_t$</td>
<td>0.0108</td>
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<td>2.9618</td>
<td>16.5138</td>
<td>0.0043</td>
<td>0.0564</td>
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<tr>
<td>$a$</td>
<td>0.0055</td>
<td>0.0095</td>
<td>0.7590</td>
<td>5.3090</td>
<td>-0.0294</td>
<td>0.0421</td>
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<tr>
<td>$b$</td>
<td>-0.0195</td>
<td>0.0098</td>
<td>-0.9843</td>
<td>6.0627</td>
<td>-0.0659</td>
<td>0.0103</td>
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<tr>
<td>$c$</td>
<td>0.0003</td>
<td>0.0000</td>
<td>0.4300</td>
<td>1.6740</td>
<td>0.0002</td>
<td>0.0003</td>
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<td>$\alpha_0$</td>
<td>1.88E-6</td>
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<td>$\alpha_1$</td>
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<td>$\beta$</td>
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<td>$\gamma$</td>
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<td>0.6194</td>
<td>2.4582</td>
<td>1.1576</td>
<td>1.2284</td>
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NMLE estimation:

Table: NMLE estimation for SP500 return rates

<table>
<thead>
<tr>
<th>Model</th>
<th>BS-</th>
<th>I-CTS</th>
<th>MJ-</th>
<th>VG-</th>
<th>CTS-</th>
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<tbody>
<tr>
<td>β</td>
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<td>0.0000</td>
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<td>λ</td>
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<td>γ</td>
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<td>1.1740</td>
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<tr>
<td>μₜ</td>
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<td>0.0003</td>
<td>0.0004</td>
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<td>σₜ</td>
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<td>zₜ</td>
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<td>0.9903</td>
<td>0.9990</td>
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<td>0.9931</td>
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</table>
### NMLE estimation:

<table>
<thead>
<tr>
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<th>VG-</th>
<th>CTS-</th>
</tr>
</thead>
<tbody>
<tr>
<td>μ</td>
<td>0</td>
<td>0.2356</td>
<td>0.1511</td>
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<tr>
<td>σ</td>
<td>1.0000</td>
<td>-</td>
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<tr>
<td>λ_J</td>
<td>-</td>
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<tr>
<td>μ_J</td>
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<td>σ_J</td>
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<td>G</td>
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## RNVR under NMLE

**Table: Risk neutral model under NMLE**

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## RNVR under NMLE:-cont’d

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**Sequential Bayesian learning approach**

**Table: Estimation with SBLA (confidence interval)**

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Sequential Bayesian learning approach

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Table: SBLA estimation for SP500 return rates

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Some figures on Bayesian learning
Some figures on Bayesian learning

### β

- Year: 2009 to 2013
- Values range from 0.7 to 0.9

### γ

- Year: 2009 to 2013
- Values range from 0 to 2

### Persistence

- Year: 2009 to 2013
- Values range from 0.8 to 1.0
Some figures on Bayesian learning
Some figures on Bayesian learning
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Panel A. RMSE Ratio of NMLE method in 2009-2012

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Panel B. RMSE Ratio of SBLA method in 2009-2012

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Evidence from options: Table IV-RMSE and ratio

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*Panel C. Ratio of NMLE method to Benchmark RMSE in 2005-2012.*

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*Panel D. Ratio of SBLA method to Benchmark RMSE in 2005-2012.*

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1 Introduction

2 Dynamic Model

3 Econometric Methodology

4 Option Valuation

5 Empirical Research

6 Conclusion and Summary
Summary: Dynamic Model

- This paper describes a new dynamic discrete-time Lévy process for option valuation, in which the latent state variables are modeled by GARCH dynamics and Lévy processes. This model can jointly capture the important stylized facts observed on assets, including time-changed volatility, stochastic jump behavior and leverage effect, which are well-documented in most financial literature.

- LIGM can be estimated by NMLE under time series framework (through fast Fourier transformation), or sequentially updated with SBLA through Monte Carlo simulation under a dynamic state space model.
Summary: Option Valuation

- We obtain an equivalent risk-neutralized pricing model based on local RNVR (with the moment generating function of Lévy innovations), and we explicitly show the transformation of all parameters and state variables under two EMMs for our dynamic models.

- Our theoretical study shows that the risk neutral parameters and states in jump models have changed a bit different from Gaussian situation. We find that, the equivalent risk neutral pricing model of non-Gaussian GARCH dynamics shows more non-normality, bigger leverage effect, higher volatility, longer persistence and stronger jump intensity.
Summary: Empirical Research

We first estimate our non Gaussian models by NMLE, in which the density of Lévy innovation is computed by FFT through their characteristic function (separately and jointly). We also develop an equivalent three-dimension state space and apply SBLA to calibrate the model, which acts as a new discrete-time dynamic Lévy process.

Based on the empirical research, we find, SBLA improves the option valuation ability of our dynamic model significantly. It also shows that, the infinite jump activity models (VG and CTS) outperform other two models (BS and MJ) in most of options. Furthermore, CTS model which describes a kind of infinite pure jump activity has the best performance totally both on NMLE and SBLA.
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